Lossy CSI-FISH:

a practical and provable secure isogeny-based signature

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Isogeny problem over elliptic curves supposed hard for quantum computers

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2019 - First efficient isogeny-based **digital signature: CSI-FiSh**

Problem: provable security of CSI-FiSh is rather weak (non-tight proof)

TIGHTNESS OF SECURITY PROOFS

An attacker able to break a cryptosystem **CS** with success probability $2^{-\delta_{CS}}$

can solve the hard problem **P** with success probability $2^{-\delta}$, where $2^{-\delta} \leq 2^{-\delta_{CS}}$

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Example - CSI-FiSh

- $2\delta_{\text{CS}} + \log_2 Q_{\text{RO}} = \delta$ <u>(classical attacker)</u>
- Best know algorithm for solving **P** has $\delta = 128$
- $2\delta_{\text{CS}} + \log_2 Q_{\text{RO}} = \delta \geq 128 \Rightarrow \delta_{\text{CS}} \geq (128 - \log_2 Q_{\text{RO}})/2$
- Assuming a rather modest $\log_2 Q_{\text{RO}} = 40$ we have $\delta_{\text{CS}} \geq (128 - 40)/2 = 44$

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Increasing the parameters would increase δ (tradeoff with efficiency), but **CSI-FiSh is specific to one set of parameters (CSIDH-512)!**

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A **better security proof** was needed

OUR CONTRIBUTION: LOSSY CSI-FISH

We propose a **new signature scheme**, Lossy CSI-FiSh, which is

- tightly secure under a **decisional variant of the isogeny problem;**
- proof of security holds also for quantum attackers;
- it is almost as efficient as CSI-FiSh
  - same signature size,
  - public key twice as large,
  - runtime for signing and verifying is (at most) twice as slow.

How? By means of a new lossy identification protocol.

ROADMAP

1. Digital signatures and the Fiat-Shamir transform
2. What is a lossy identification protocol?
3. Our CISDH-based lossy identification protocol
4. Why a lossy identification protocol?
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The digital signature DS is **secure** if **an attacker knowing pk (but not sk)** has **negligible success probability** in producing a pair (σ^*, m^*) s.t.

$$\text{Verify}(\text{pk}, \sigma^*, \text{m}^*) = 1$$

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It has been widely used since its introduction (Crypto 1986)

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$$\mathsf{ID} = (\mathsf{IGen}, \mathsf{P} = (\mathsf{P}_1, \mathsf{P}_2), \mathsf{V})$$

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Required properties

- Correctness
- Honest-Verifier Zero-Knowledge
- High Min-Entropy
- Perfect Unique Response
- 2-Special Soundness

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- Statistical Lossy Soundness $\rightarrow$ An **unbounded** adversary $\mathscr{A}$ produces a **valid transcript** for $\mathrm{pk}_{\mathrm{ls}}$ with probability ϵ_{ls}.

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- Indistinguishability of Lossy Statements

$$(\mathsf{pk}_{ls}, \cdot\,) \leftarrow \mathsf{LossyIgen}(1^{\lambda})$$

$\mathsf{Adv}^{\mathsf{lossy}}_{\mathscr{B}}(\lambda)$ in distinguishing real and lossy public keys is **negligible**

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THE CSIDH SETTING

- G finite abelian group
- X finite set

G acts **freely** and **transitively** on X

$$\begin{aligned} \star : G \times X &\to X \\ (g, \mathsf{X}) &\mapsto g \star \mathsf{X} \end{aligned}$$

- $1_G \star \mathsf{X} = \mathsf{X}$;
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Computing **class numbers** of quadratic orders requires **subexponential complexity.**

CSI-FiSh performed a **(record) class group computation**

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Decisional CSIDH (D-CSIDH) problem - distinguish between the distributions

$$(E, H, \mathfrak{g}^a \star E, \mathfrak{g}^a \star H) \text{ and } (E, H, E', H')$$

where $E, H, E', H' \leftarrow X, a \leftarrow \mathbb{Z}_N$.

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$$E_0 \xrightarrow{\mathfrak{g}^a} E$$

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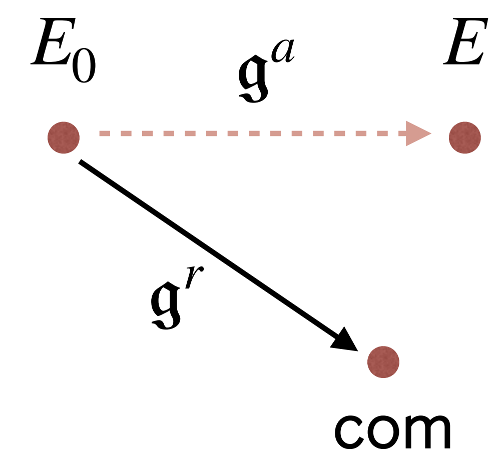

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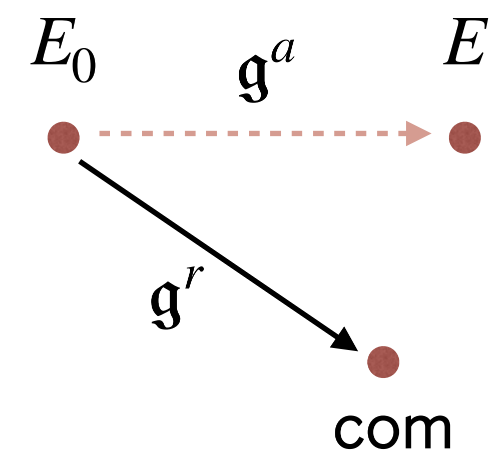

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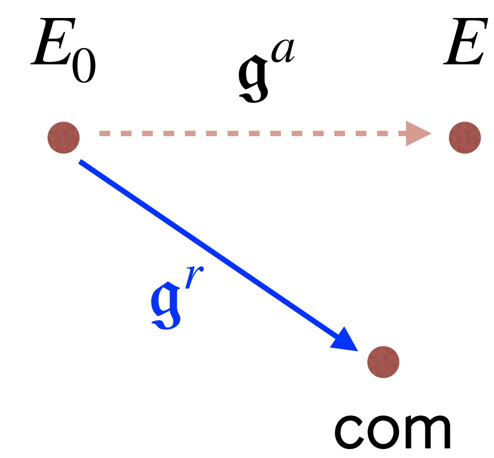

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(ch $= 0$) resp $:= r$,

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pp $= (p, \mathfrak{g}, N, E_0 \in X)$

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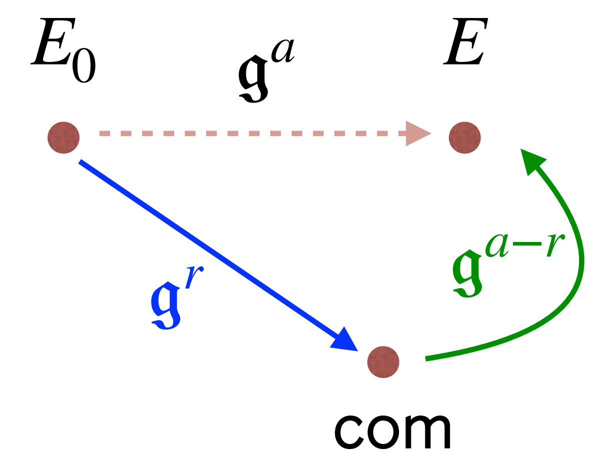
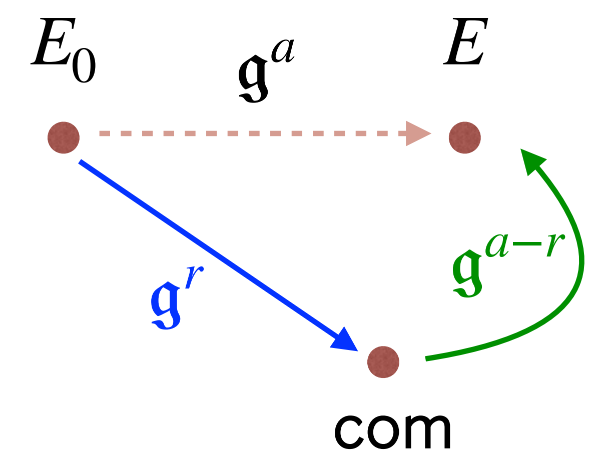


Prover		**Verifier**
$r \leftarrow \mathbb{Z}_N$, com $:= \mathfrak{g}^r \star E_0$	$\xrightarrow{\text{com}}$	
	$\xleftarrow{\text{ch}}$	ch $\leftarrow \{0,1\}$
(ch $= 0$) resp $:= r$, (ch $= 1$) resp $:= a - r$	$\xrightarrow{\text{resp}}$	

CSI-FISH ID

$\mathsf{pp} = (p, \mathfrak{g}, N, E_0 \in X)$

$\mathscr{R}_{\mathsf{CSI-FiSh}} = \{(E, a) \,|\, E = \mathfrak{g}^a \star E_0\}$



Prover | **Verifier**

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$\xleftarrow{1/0}$

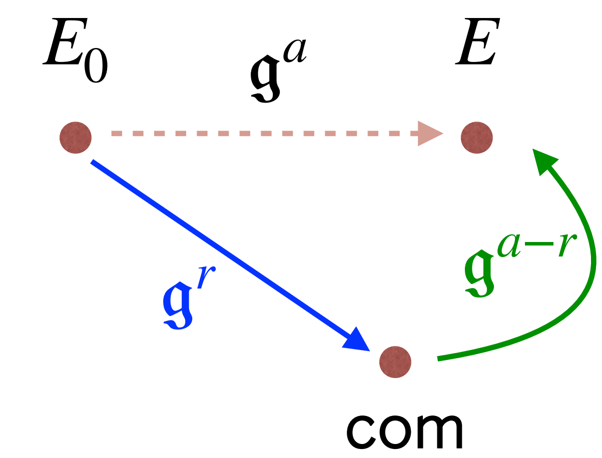
(ch = 0) com $= = \mathfrak{g}^{\text{resp}} \star E_0$

(ch = 1) $E = = \mathfrak{g}^{\text{resp}} \star$ com

OUR LOSSY ID

$\mathsf{pp} = (p, \mathfrak{g}, N, E_0 \in X)$

$$\mathscr{R}_{\text{Lossy CSI-FiSh}} = \{((E_1^{(0)}, E_2^{(0)}, E_1^{(1)}, E_2^{(1)}), a) \mid E_i^{(1)} = \mathfrak{g}^a \star E_i^{(0)},\ i = 1,2\}$$



Prover | **Verifier**

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$\xleftarrow{\text{ch}}$ $\mathsf{ch} \leftarrow \{0,1\}$

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$(\mathsf{ch} = 0)\ \mathsf{com} == \mathfrak{g}^{\mathsf{resp}} \star E_0$

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OUR LOSSY ID

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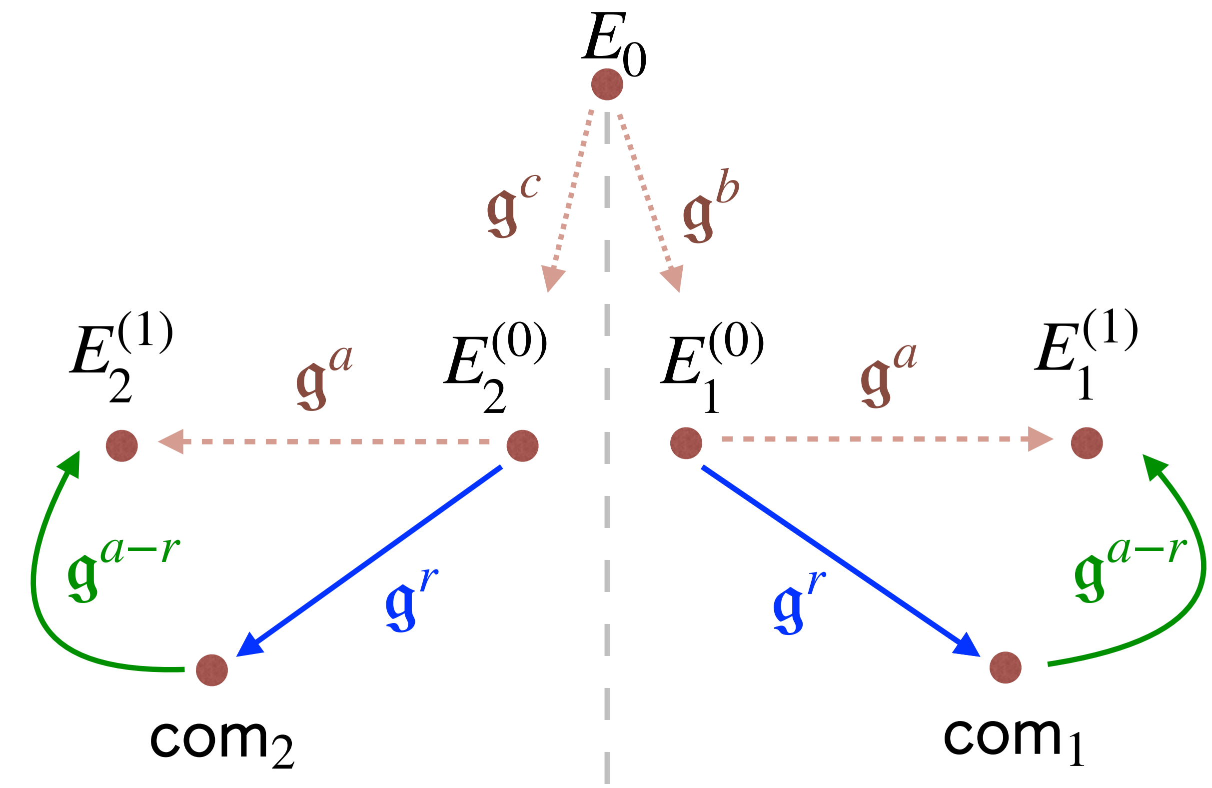

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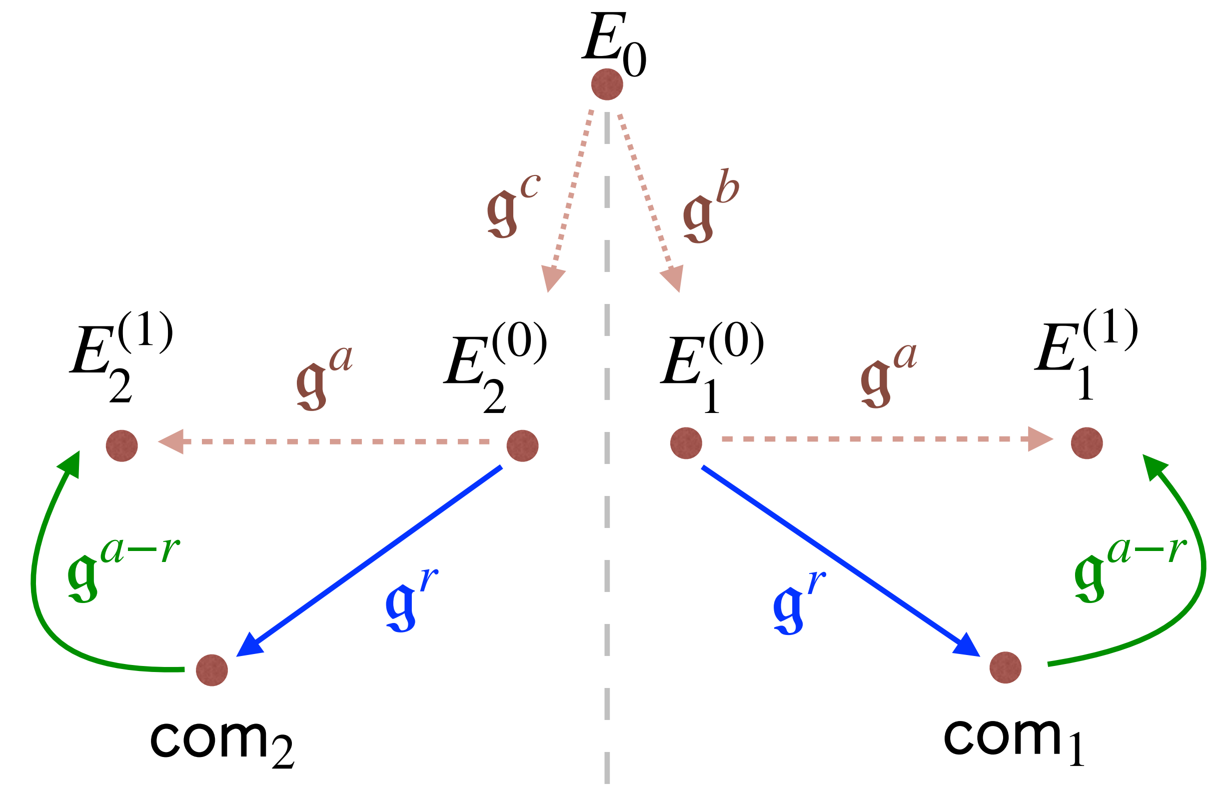

Prover | **Verifier**

$r \leftarrow \mathbb{Z}_N,\ \mathsf{com}_i = \mathfrak{g}^r \star E_0^{(i)}$

$\xrightarrow{\mathsf{com} = (\mathsf{com}_1, \mathsf{com}_2)}$

$\mathsf{ch} \leftarrow \{0,1\}$

$\xleftarrow{\mathsf{ch}}$

$(\mathsf{ch} = 0)\ \mathsf{resp} := r,\ (\mathsf{ch} = 1)\ \mathsf{resp} := a - r$

$\xrightarrow{\mathsf{resp}}$

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$(\mathsf{ch} = 0)\ \ \mathsf{com}_i == \mathfrak{g}^{\mathsf{resp}} \star E_i^{(0)}$

$(\mathsf{ch} = 1)\ \ E_i^{(1)} == \mathfrak{g}^{\mathsf{resp}} \star \mathsf{com}_i$

OUR LOSSY ID

Properties

- Correctness
- Honest-Verifier Zero-Knowledge
- High Min-Entropy
- Perfect Unique Response

OUR LOSSY ID

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- Statistical Lossy Soundness

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An **unbounded** adversary $\mathscr{A}$ produces a **valid transcript** for $\mathsf{pk}_{\mathsf{ls}}$ with probability ϵ_{ls}.

$$\epsilon_{\mathsf{ls}} = \frac{1}{2} + \frac{1}{2N}$$

OUR LOSSY ID

Properties

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- Honest-Verifier Zero-Knowledge
- High Min-Entropy
- Perfect Unique Response
- Statistical Lossy Soundness ($\epsilon_{\mathsf{ls}} = 1/2 + 1/2N$)
- Indistinguishability of Lossy Statements

$$\downarrow$$

$(\mathsf{pk}_{\mathsf{ls}}, \cdot\,) \leftarrow \mathsf{LossyIgen}(1^\lambda)$

$\mathsf{Adv}^{\mathsf{lossy}}_{\mathscr{B}}(\lambda)$ in distinguishing real and lossy public keys is **negligible**.

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Real public key: $(E_1^{(0)} = \mathfrak{g}^b \star E_0, E_2^{(0)} = \mathfrak{g}^c \star E_0, \mathfrak{g}^a \star E_1^{(0)}, \mathfrak{g}^a \star E_2^{(0)})$

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Lossy public key: $(E_1^{(0)} = \mathfrak{g}^b \star E_0, E_2^{(0)} = \mathfrak{g}^c \star E_0, E', H')$

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Lossy public key: $(E_1^{(0)} = \mathfrak{g}^b \star E_0, E_2^{(0)} = \mathfrak{g}^c \star E_0, E', H')$

Decisional CSIDH (D-CSIDH) problem - distinguish between the distributions

$$(E, H, \mathfrak{g}^a \star E, \mathfrak{g}^a \star H) \text{ and } (E, H, E', H')$$

where $E, H, E', H' \leftarrow X, a \leftarrow \mathbb{Z}_N$.

ROADMAP

1. Digital signatures and the Fiat-Shamir transform ✓
2. What is a lossy identification protocol? ✓
3. Our CISDH-based lossy identification protocol ✓
4. Why a lossy identification protocol?
5. Security and efficiency of Lossy CSI-FiSh

WHY A LOSSY IDENTIFICATION PROTOCOL?

<u>Theorem</u> (Kiltz, Lyubashevsky, Schaffner - 2018)

Let ID be a **lossy identification protocol** (correct, Honest-Verifier Zero-Knowledge, α bits of min-entropy, Perfect Unique Response, ϵ_{ls}-statistical lossy soundness, indistinguishability of lossy statements), then

$$\mathsf{Adv}_{\mathscr{A}}^{\mathsf{su-cma}}(\lambda) \leq \begin{cases} \mathsf{Adv}_{\mathscr{B}}^{\mathsf{lossy}}(\lambda) + (Q_H + 1) \cdot \epsilon_{\mathsf{ls}} + 2^{-\alpha+1} + \mathsf{Adv}_{\mathscr{D}}^{\mathsf{PRF}}(\lambda) & \text{(ROM)} \\ \mathsf{Adv}_{\mathscr{B}}^{\mathsf{lossy}}(\lambda) + 8(Q_H + 1)^2 \cdot \epsilon_{\mathsf{ls}} + 2^{-\alpha+1} + \mathsf{Adv}_{\mathscr{D}}^{\mathsf{PRF}}(\lambda) & \text{(QROM)} \end{cases}$$

and $\mathsf{Time}(\mathscr{B}) = \mathsf{Time}(\mathscr{D}) = \mathsf{Time}(\mathscr{A}) + Q_H \approx \mathsf{Time}(\mathscr{A})$.

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CLASSICAL SECURITY OF LOSSY-CSI-FISH

We focus on CSIDH-512 parameters.

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				Lossy CSI-FiSh		CSI-FiSh
S	t	u	$\|\sigma\|$	$\|\mathsf{pk}\|$	Bits of security	$\|\mathsf{pk}\|$
1	74	16	2405B	256B	127	64B
3	43	14	1403B	512B	126	192B
7	30	16	983B	1024B	125	448B
15	25	13	822B	2048B	124	960B
2^6-1	17	16	564B	8.2KB	122	4KB
2^8-1	14	11	468B	32.8KB	120	16.3KB
$2^{10}-1$	12	7	404B	131KB	118	65.5KB
$2^{12}-1$	10	11	339B	524KB	116	262KB
$2^{15}-1$	8	16	274B	4MB	113	2MB

QUANTUM SECURITY OF LOSSY-CSI-FISH

We focus on CSIDH-512 parameters.

			Conservative variant			Optimistic variant		
S	u	$\vert \mathsf{pk} \vert$	t	$\vert\sigma\vert$	Bits of security	t	$\vert\sigma\vert$	Bits of security
1	16	256B	64	2080B	55	74	2405B	63
3	14	512B	37	1208B	54	43	1403B	62
7	16	1024B	26	852B	53	30	983B	61
15	13	2048B	21	691B	52	25	822B	60
2^6-1	16	8.2KB	15	497B	50	17	564B	58
2^8-1	11	32.8KB	12	401B	48	14	468B	56
$2^{10}-1$	7	131KB	10	337B	46	12	404B	54
$2^{12}-1$	11	524KB	9	305B	44	10	339B	52
$2^{15}-1$	16	4MB	7	240B	41	8	274B	49

EFFICIENCY OF LOSSY-CSI-FISH

Costs are dominated by the computation of class group actions:

- **Key Generation:** $2S + 2$ (S in CSI-FiSh)
- **Signing:** $2S$ (S in CSI-FiSh)
- **Verifying:** $2S$ (S in CSI-FiSh)

Estimated running times

(S, t, u)	Key Gen	Sign	Ver
$(2^{15} - 1, 7, 16)$	56m	800ms	800ms
$(2^{3} - 1, 28, 16)$	920ms	3s	3s

Thanks for your attention

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